INTEGRATED ~~WAREHOUSE~~ AND PRODUCTION SYSTEM: STOCK ASSIGNMENT AND ORDER PICKUP

by

KARUNESH AGARWAL

DEPARTMENT OF INDUSTRIAL AND MANAGEMENT ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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INTEGRATED WAREHOUSE AND PRODUCTION SYSTEM -

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CERTIFICATE

It is to certify that the work contained* in the thesis entitled " INTEGRATED WAREHOUSE AND PRODUCTION SYSTEM - STOCK ASSIGNMENT AND ORDER PICKUP" by Mr. Karunesh Agarwal has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

(A. K. Mittal)
Professor
Industrial and Management Engineering Department
Indian Institute of Technology
Kanpur 208 016

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* Most of the material in chapter I and chapter II in this thesis is common with the material in chapter I and chapter II, of thesis entitled "INTEGRATED WAREHOUSE AND PRODUCTION SYSTEM - SYSTEM DESIGN AND IMPLEMENTATION", also being submitted for M.Tech degree by Mr. Rohit Bansal, Roll No. 9311421.

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A strongly connected IME graph composed of twenty-one nodes rendered my stay at IIT-K a memorable one. I had the privilege to be acknowledged the proprietor of the *saraih* tree, the most prominent in the forest. The innumerable discussions at *saraih* ranging from psychoanalysis to optimization techniques, contributed significantly to my personal and professional development. My appreciation to the *saraih* members : Srinivas, Sriram, Rajkumar Jain and Rajeev Seth for what they are and would be.

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ABSTRACT

In this dissertation an attempt has been made to solve the practical problem of Stock Assignment Problem in a warehouse for a large automobile plant. Twenty such problems are identified on the basis of factors such as number of items to be stored, number of bins of an item to be stored, choice of equipment and mode of retrieval. The cases discussed are from the very simple one, that is, to store one bin of an item without any choice of equipment to that of the most general case, of storing more than one bin of different items when choice of equipment is present and the equipment's can handle more than one bin at a time. The structure of these problems is examined and the problems are structured as combinatorial optimization problems.

These problems were found to be similar to the standard problems such as Maximum Weighted Clique Problem(MWCP), Time Tabling Problem, Transportation Problem, Generalized Quadratic Assignment Problem(GQAP), Traveling Salesman Problem(TSP) and the k-TSP.

Most of these Problems are NP-Hard and hence polynomial time solutions to these problems are not expected. However some efficient heuristics, based on the known heuristics for the above problems are developed.

Order picking problem is discussed and some possible solution methodologies for various cases of order picking are also suggested.

CONTENTS

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Karunesh

(Karunesh Agarwal)

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CHAPTER I
INTRODUCTION

1.1 Introduction

The work on this dissertation is motivated by the warehouse problem of a major automobile company. The company has recently established a new plant and the facilities are under construction. In this company vehicles are assembled on a assembly line, which run continuously. Frame(chassis) of the vehicle is mounted on the first station of the assembly line and it leaves the system with the vehicle ready for despatch, from the last station. Parts and subassemblies are stored in five different types of bins in central warehouse which is at a distance from the assembly line.

Reddy J.M.[38] worked on linking the warehouse with production line by optimizing number of fork lift trucks and trolleys to be used for transferring parts from the central warehouse to the assembly stations for the production purpose. In this dissertation we will concentrate on the storage and retrieval of the materials in the warehouse, which will involve selection of the location to store the items, the assignment of the equipment for storing, allocation of the items to the equipment and the planning of the route (tour) for the equipment when more than one bin can be stored, at the same time. Similarly the retrieval requests have to be assigned to the retrieval equipments and route of the pick up equipment has to be planned.

In practice the aim is to design an on-line operating system for the smooth and optimized running of the warehouse, which will link the incoming material storage with the demand at the production line.

1.2 Warehousing

In the past few years, the field of warehousing has begun to receive considerable attention. Now the warehouse manager is required to increase customer service, reduce inventories, increase productivity, handle large number of stock keeping units, and improve space utilization.

The functions performed by the warehouse are:

[1] Receiving the goods from the stores.

[2] Storing the goods until they are required

[3] Picking the goods when they are required.

[4] Shipping the goods to the appropriate user.

Warehouse planning is not simply pouring a concrete slab, installing some racks, and tilting up some walls. Nor it is a static one time activity. the changing dynamic environment in which warehouses are planned quickly renders existing plan obsolete. Therefore we need an on-line dynamic system in which warehouse planning is a continuous activity and the existing plan is constantly being scrutinized and molded to meet anticipated requirements. A successful warehouse maximizes the effective use of the warehouse resources while satisfying customer requirements.

The following objectives must be met for a warehouse to be successful:

[1] Maximize the effective use of space

[2] Maximize the effective use of equipment

[3] Maximize the effective use of labor

[4] Maximize the accessibility of all items

[5] Maximize protection of all items.

In warehousing a distinction is made between a finished goods warehouse and a raw materials storeroom. The only true distinction between the two are the sources from which the goods are received and the user to which the goods are shipped. A raw materials storeroom receives goods form the outside source, stores the goods, picks the goods, and ships the goods to an inside user. A finished goods warehouse receives the goods from an inside source, stores the goods, picks the goods, and ships the goods to an outside user. Likewise, an in process inventory warehouse receives goods from an inside source, stores the goods, picks the goods, and ships the goods to an inside user. Our problem is mainly concerned with the raw materials storeroom and in process inventory warehouse.

Warehousing for the purpose of commercial gain is at least as old as recorded history. In early writing, man was described as having stored excess foods and kept animals for emergency surplus. As civilization developed , local warehouses were introduced. When major trade point were introduced during middle ages, warehousing was established to store the shipped items. All the developments stressed more upon the warehouse location and connection with the external sources and demand points. But as warehousing systems advanced from local warehouses during the middle ages to multimillion dollar facilities, more attention was being paid towards what is going inside the warehouse.

Computer directed warehousing systems using stacker cranes and palletized loads have revolutionized the design and capacity of large capacity, high volume storage facilities. Now more attention is being paid towards saving labor costs, high floor space utilization, improved material flow, improved inventory control, and a lower incidence of misplacement or theft. Maximized benefits of such a system are dependent upon the optimal design of the system, that is, operating cost of warehouse(storage and retrieval costs and time). [Tompkins J. and Smith J.D.[43]].

1.3 Literature Review (review of previous work)

There have been quite a number of studies on various aspects of automated warehousing systems. We shall review the literature in four groups :

1.3.1 Optimal storage assignment problem

There are very few research papers in the management science or operations research literature dealing with the optimal storage assignment in the warehouse. A very limited number of books, articles, and pamphlets dealing with the subject are available, that too generally in non technical manner.

The location of stock in a warehouse so as to minimize the cost of assembling or handling orders is an important problem in industrial logistics (Neal 1962,1967)[33]. The objective is generally to minimize the man hours required to pick the orders and transport them to the shipping area. The optimal arrangement of stock will depend

upon the order picking method, turnover rates, product space requirements, and the demand relationship among the products.

Wilson (1977)[45], discussed some results for a simple out and back picking method. The cube-per-order index (COI) criterion for assigning inventory items to locations within the warehouse was proposed by Heskett (1964)[18]. The COI for an item is simply the quotient of the space which must be allowed for the item and the order frequency for the item. Recently, Kallina (1976)[25], showed that the COI rule produces an optimal location selection, subjected to certain assumptions.

One of the outstanding series of works in these are that of Hausman, Schwarz and Graves, (1976)[17]. They compared the operating performance of the three storage assignment rules: random assignment, full turnover based assignment and class based turnover assignment. They studied the effect of various storage and retrieval policies on crane travel time per storage and retrieval. They concluded that significant reductions in crane travel time (and distance) are obtainable from turnover based rules.

In the extension to this paper (1977)[15], they included interleaving, i.e. the sequencing of storage and retrieval requests. The paper compares the operating performance of several storage assignment/interleaving policies. It is shown that significant reduction in crane travel time and distance are obtainable in some real world situations via the proposed storage assignment/ interleaving policies.

In another paper(1978)[40], using a computer simulation the scheduling policies reported in the pervious papers for deterministic environment have been examined in a stochastic environment and the results are extended to the conditions of imperfect information.

Rosenblatt M.J. and Roll Y. (1984)[39], developed a search procedure for finding a global optimal solution for a specific formulation of the warehouse design problem. In this formulation they considered three types of costs : cost associated with the initial investment, a storage cost and costs associated with the storage policy. They tested several policies ranging from completely random storage to completely grouped storage on a simulated model of a warehouse. They showed that, in certain situations,

considerable administrative and other advantages can be realized by group storage with only slight increase in the required warehouse spaces. The proposed procedure enables one to obtain parameters for the design of warehouses so that storage needs are met in a optimal way cost wise. Physical characteristics as well as storage policies are combined to give the best performance.

Linn and Wysk (1984)[31], compared the effects of several control policies regarding stacker movement, sequencing rules, and storage location assignment on the simulated model of an automated storage/retrieval system. After a extensive number of simulation runs and testing various settings for each of the policies, they came up with several guidelines regarding each class of control policies.

Jay M. Jarvis and Edward D. McDowell (1991)[24] provided a basis for locating product in an order picking warehouse such that order picking time will be minimized. They showed that if the aisles are not symmetrically located about the dock, then simply assigning the most frequently picked items to the nearest aisles will not necessarily minimize the average travel distance. A heuristic based on these conditions was then developed.

Moon-Kyu Lee(1992)[32] dealt with a man on board automated storage / retrieval system where each customer order consist of number of different items and is picked one at a time. For the system the problem examined was to allocate storage allocation dedicated to the items so that total travel time required to pick up all the given orders per period is minimized. He developed a heuristic based on the group technology concept considering both order structure and frequency. Through the heuristic, close relationships between items were identified from the order structure, and then based on the relationships the items are compelled to be stored closely in the storage rack.

1.3.2 **Optimal order picking problem**

The order picking problem is the complementary of the above problem where orders are received and the optimal sequence for picking these orders is to be found. Phillips(1977) [35] used the network concept as a basic analytical technique solving

order picking problems. In his technique, he considered the nodes of network as locations of items while the distance between two locations is presented as an arc value. Chisman (1977) [7] proposed several methods for picking orders. Orders are divided into three groups: batch carrier, line batch and order picking batch. In carrier batch, geographic similarity of the order's is the main criterion for assigning the orders to such a group. A line-batch group includes all the items which are placed on one shipping line prior to loading on a truck, while the order-picking-batch includes all the items placed on one pallet as a bin-picker passes through the warehouse. Chisman(1977) [7] suggested two methods for bin picking : SN-picking (Stock Number picking) and BL-picking (Bill-of-loading picking). He stated that the choice between SN-picking and BL-picking is a matter of whether one wants to or finds it cheaper to sort out BL's at the bins or in the shipping line. The solution to this problem was obtained by using a cluster traveling salesman algorithm developed by Chisman (1975) [6].

Elsayed E. A. (1981) [8], presented four heuristic algorithms for handling orders in automatic warehousing systems. The algorithms select the orders that will be handled in one tour in order to minimize the total distance traveled by the S/R machine within the warehouse system. Computer programs are developed for the four algorithms and the optimal tours are found by using the traveling salesman algorithms. Optimal or near optimal solutions are found for the handling problem The conclusion reached was that optimum policy is problem dependent. Thus, he suggested, for a given problem it is better to evaluate all the policies and pick the one that provides the best result.

Elsayed E. A. and Stern R. G. (1983) [9], presented some new algorithms for processing a set of orders in automated warehousing systems. The proposed algorithms will process the orders by grouping the orders according to some criteria developed by the authors. They proposed four *seed selection* criteria, three *order congruency* criteria and two *order addition* criteria. Thus, they suggested 4*3*2 = 24 possible algorithms for the order picking problem. The traveling salesman algorithm is then utilized to determine the optimal distance traveled within the warehouse for every

group of orders. Comparisons of the performance of the proposed algorithms are also presented. The proposed algorithms proved to be data dependent and were sensitive to the capacity of the picking vehicle.

Hwang H, Wonjang B. and Moon Kyu Lee (1988) [23], presented heuristic algorithms for batching a set of orders such that the total distance traveled by the order picking machine is minimized. These algorithms were based on the cluster analysis and their efficiency and validity were illustrated through computer simulation The results obtained were found to be better than those from the previous studies.

Goetschalckx M. and Ratliff H. D. (1988) [14], presented a special case in which items have to be retrieved manually from both sides of a wide aisle and deposited on a vehicle which travels on the center line of the aisle. The picker will stop the vehicle, pick and load the cases of items onto the vehicle and then drive to the next stop. They presented an algorithm to determine the optimal number and location of stops and specify the items to be picked at each stop.

Hwang H. and Moon-Kyu Lee (1988) [22], dealt with order processing problem in a man-on-board automated storage and retrieval system. They presented new heuristic algorithms based on cluster analysis. The algorithms process the orders by batching them according to the value of similarity coefficient which is defined in terms of attribute vectors. To find the minimum travel time for each batch of orders, the traveling salesman algorithm is employed. The results obtained through computer solution indicates that some algorithms developed performed substantially better than those of the previous studies.

1.3.3 Throughput maximization problem

Azaidivar F. (1986) [1], in his paper attempted to determine the maximum number of storage and retrieval requests that can be handled by automated warehousing systems under physical and operational constraints. He formulated the system as a stochastic constrained optimization problem and developed an algorithm for its solution. The constraints involved limits on the average waiting time for retrieve requests and the

maximum queue length for the items waiting to be stored. Due to the stochastic nature of the operation of the system, a simulation model was constructed to evaluate the system for various values of its parameters.

Azaidivar F. (1989) [2], presented a method for the optimum allocation of the resources between the Random access spaces and Rack spaces so that the overall throughput capacity of the warehouse is maximized. He formulated and solved the problem as a stochastic optimization problem.

1.3.4 Warehouse design problem

Hwang H. and Chang S. Ko (1988) [21], suggested a Multi-aisle S/R machine system (MASS) which can substantially reduce high initial investment cost which is a major reason for the low popularity of AS/RS in manufacturing companies. The objective of their study was mainly related to the design aspect of MASS. With a travel time model developed they determined the average travel time of the S/R machine. They proposed rack-class-based storage assignment procedure and class selection procedure to find the minimum number of S/R machines required and identified the number of aisles each S/R machine serves. The procedures applied to example problem showed that MASS is effective in reducing initial installation cost, provided that the pallet demand are relatively low.

Park Young H. and Dennis B. Webster (1989) [46], developed an optimization procedure to aid a warehouse planner in the design of selected three dimensional, palletized storage systems. All the alternatives were compared in the overall model while simultaneously considering the following factors : control procedures, handling equipment movement in an aisle, storage rules, alternative handling equipment, input and output pattern for product flow, storage rack structure, component costs and the economics of each storage system. The above review reveals that there is an abundance of potential research areas that deserve further study.

1.4 Present Work

In this dissertation, as we have stated, we will deal with the storage assignment problem for a large automobile plant. In chapter II first we will describe the problem situation and then we will deal with the concept of correlated assignment, that is similar items, which are frequently requested together should be assigned to the nearby locations. Assumptions made while formulating the problem and a system of classification for the problem are also mentioned.

The strategy followed is that we will start with the simplest possible case and then we will go on imposing the constraints and complicating the problem environment towards a real life situation. In chapter III we will deal with the case of single bin storage. The specific structure of the problem will be identified and then the mathematical formulation (0-1 integer program) will be done. The solution methodologies for the formulated problems will also be discussed along with them.

Likewise in chapter IV the case of storing more than one bins at a time, when equipment can handle only one bin at a time will be discussed. Various possible cases will be identified and then mathematical formulations along with the solution methodologies will be discussed for them. The problems identified in this chapter are the special case of transportation problem, maximum weighted clique problem and quadratic assignment problem. Heuristics as well as exact methods will be discussed for all the cases.

In chapter V we will deal with the problem of storing more than one bin at a time, when equipment can handle more than one bin at a time. Here also the specific structure of the problem will be identified, which will be followed by mathematical formulations and solution methodologies.

In chapter VI, we shall discuss the order picking problem for all the possible cases.

In chapter VII, we shall summarize and suggest few directions for the future work.

CHAPTER II

PROBLEM DESCRIPTION & CONCEPT OF CORRELATED ASSIGNMENT

2.1 Description of the problem

In the section 1.1 we have described the motivation for this work . As stated, the work on this dissertation was motivated by the warehouse planning problem of a major automobile company. The company has recently setup a new plant and the facilities are under construction.

In this company, vehicles are assembled on assembly line, which runs continuously. Frame (chassis) of the vehicle is mounted on the first station of the assembly line, and it leaves the system with the vehicle ready for despatch, from the last station. Parts and subassemblies are assembled on the chassis at relevant assembly stations as the conveyer of the assembly line moves. These parts and subassemblies are stored in five types of standard size bins in central warehouse which is at a distance from the assembly line. So these parts are to be transferred to the required assembly stations for the assembly purpose by forklifts and trolley attached trucks. Whenever such a forklift or trolley attached truck comes to central warehouse with an order, it is a retrieval request that is to be processed inside the warehouse. As forklift can carry only one bin at a time, the retrieval request associated with it is of a single bin only. But in case of trolley attached trucks, the retrieval request is a menu, comprising of several bins of different items to be retrieved. Similarly at the receiving end of the warehouse, whenever the in-process inventory or raw material inventory from an external source is received to be stored, it is a storage request to be processed inside the warehouse. The storage request can be a single bin storage request or it may also be in the form a menu. With the advent of information technology, it is possible to plan a on-line decision support system for the optimized and smooth operations of the

warehouse. This decision support system should be capable of handling the problems described in the next section.

2.1.1 The stock assignment problem

Whenever a storage request reaches the warehouse, the system should optimally locate the items in the available locations. Optimality criterion will include the distance traveled while storing the item as well as the retrieval distance associated with that particular location. In case where retrieval is done in batches, the optimality criterion will also include the desirability of storing the items close to the items with which they are frequently retrieved so that the tour length of the retrieving equipment is minimized while retrieving the order. The decision support system should also be capable of selecting the equipment's for serving each location which has been chosen for locating the item If the equipment has capacity for simultaneously carrying more than one bin, the selection of bins for each trip and the tour has also to be planned.

2.1.2 The order picking problem

The order picking problem is the complementary of the above problem. Whenever a retrieval request reaches the warehouse, the system should decide on optimal sequence for picking these items. Here also there may be several cases. Retrieval request may be of a single bin of an item or it may be a menu comprising of several bins of an item. The pick up sequence generally is to be optimized with respect to the retrieval cost, which is a function of travel, loading, unloading cost and equipment choice.

When equipment can handle more than one bin at a time then the locations of the bins and the path (tour) of the equipment has to be decided. Further when more than one retrieval requests is received then requests can be clustered together from different orders subjected to time restraint, that is, due delivery time of the order.

2.1.3 Salient features of the production system

The company presently, produces four to six models of vehicles, but has plans of producing atleast twenty different models. Each model requires different parts or sub-assemblies, some of which are common to other models also. Each part may be required at more than one work station. Part may not be work station specific, that is, same part which is required at a particular station for a model may be required at another work station for another model. The Bill of material consisting of part identification and number of part required per vehicle for each model is assumed to be available. Further the requirement of the parts at different work stations can be computed from the production schedule. We shall assume that each station has a sufficient capacity to accommodate reasonable amount of inventory. Further we will assume that a vehicle production schedule is available in advance, that is, batch sequence, type of model for each batch and quantity of items to be produced.

The retrieval requests originate from the work stations. It has been observed that the trend of retrieval requests is consistent for a particular model. The problem of linking of workstations requirement with the production schedule and order pick up schedule has been solved in Reddy [38]. In his thesis Reddy [38], has developed algorithms to link the production schedule (Batch sequences) into the requirements of part inventory through the concept of due date (latest delivery time) at the work station. Further algorithms for the assignment of these required item bins to the transportation equipment (Trolleys or fork lift trucks) from warehouse to workstation and time schedule for despatch of these equipment have been developed. The end output of these algorithms is to provide an order pick up schedule (equipment wise) with latest time of filling the order at the output dock of the warehouse. This order pickup schedule will act as input to our problem.

It is observed that for a particular model, the items which are retrieved together once are retrieved together most of the times. So, from the analysis of the past pick up schedule

one can establish the likelihood of a pair of items being retrieved together. This concept is known as correlated assignment concept, and is described in the next section.

2.2. Concept of Correlated Assignment

The concept of correlated assignment is that the stock keeping units (SKU's), or bins of item, which are frequently requested together (that is, in the same order or in the same time window), should be stored together.

Note that the correlated assignment applies for strict order picking and batch picking operations. We will try to quantify the desirability of the two items to be stored together by defining a penalty function, that is dissimilarity cost.

2.2.1 Scheme for Computing Dissimilarity Cost

The dissimilarity cost can be described as the cost incurred when items which are frequently retrieved together in the same order are not stored close to each other. However, such a dissimilarity function will be specific to a particular warehouse situation. We will describe three such cases in which dissimilarity cost is calculated primarily based on distance consideration and the expected frequency of joint retrieval. However first we shall discuss the distance function for likely warehouse movement for combined retrieval.

2.2.1.1 Distance Function

The distance function between two locations i and j i.e., d_{ij} is defined as the weighted horizontal, vertical, and across the aisle distance between them .

$$d_{ij}=\{H|Xi-Xj|\hat{}a +A|Yi-Yj|\hat{}b+ V|Zi-Zj|\hat{}c\}\hat{}d \quad (2.2.1.1.1)$$

Where

H is the weightage for the horizontal distance .

V is the weightage for the vertical distance.

A is the weightage for across the aisle distance.

X_i, X_j are the x-coordinates of locations i and j.

Y_i, Y_j are the y-coordinates of locations i and j.

Z_i, Z_j are the z-coordinates of locations i and j.

a,b,c,d are the constants to determine the type of distance function, that is, rectilinear, euclidean or squared euclidean.

So if we want to include the rectilinear horizontal distance only when calculating the distance between two locations we can keep the value of H,a and d equal to one and weightages V,A,b and c will take value Zero.. Likewise these weightages take the values according to our requirement of the distance function.

The dissimilarity cost of an item stored (or to be stored) in a particular location depends upon the following factors :

[1]. The frequency of retrieval of the item to be stored with the other items already stored in the warehouse.

[2]. The type of items and the number of bins of other items stored in the warehouse.

[3]. The distance of the locations of these items from the location under consideration.

[4]. If more than one bin of the same item or different items are to be stored then the interaction between these items to be stored, which in turn will depend upon the selection of the locations for these items from the available locations.

One such illustrative function for the dissimilarity cost can be computed as shown below. The basis of this function is that :

(a) Closer the item stored, lesser the cost for dissimilarity.

(b) Higher the joint retrieval frequency, higher is the cost for dissimilarity.

2.2.1.2 Dissimilarity cost when only single bin of an item is to be stored.

In this case we have to locate one bin at one location out of available "m" locations. So this case is similar to single facility location in which we have to locate one facility at one

location out of some pre specified number of available locations. Here a dissimilarity cost will be associated with each available location. Let it be denoted by a_i.

$$a_i = \sum_{o \in S} f_o * d_{i(I_i,o)} \text{.....................} (2.2.1.2.1)$$

where

a_i = dissimilarity cost of location i

f_o = joint retrieval frequency of item o and item to be stored.

d_{ij} = distance between location i and j.

$I_{i,o}$ = Location of item o closest to the location i.

S = Set of all items in the warehouse.

This cost is computed based on the assumption that in case of simultaneous retrieval of item "o" with the item being stored in the location "i", item "o" will be picked up from the closest location in which it is stored.

2.2.1.3 Dissimilarity cost when more than one bin of same item are to be stored simultaneously .

In this case we have to locate more than one bins of same item, so we should also take into account the interaction cost of locating the items at more than one location. Let the total dissimilarity cost of locating the item at both the locations i and j be denoted by a_{ij}.

$$a_{ij} = a_i + a_j - p_{ij} \text{..................................} (2.2.1.3.1)$$

where p_{ij} is the interaction term and is computed as mentioned below.

$$p_{ij} = \sum_{o \in S} f_o * b_{ij} * \text{Maximum}[d_{i,(I_i,o)}, d_{j,(I_j,o)}] \text{.............} (2.2.1.3.2)$$

where b_{ij} = 1 if $I_{i,o} = I_{j,o}$

= 0 otherwise.

This interaction term is computed taking into consideration that in case for item "o" same location is close to both the selected locations for the item being stored in it, it has to be associated with only with one of them, which will be the location closer to it.

2.2.1.4 Dissimilarity cost when more than one bin of different items are to be stored simultaneously.

This is a case similar to the case of multifacility location in which new facilities to be located have interaction among them. So there will be a dissimilarity cost associated with each location for a particular type of item and there will be an interaction factor for two type of items being located at two locations.

Let

a_{ik} = dissimilarity cost of locating the ith item at kth location.

a_{jl} = dissimilarity cost of locating the jth item at lth location.

a_{ijkl} = dissimilarity cost of locating the ith item at kth location and jth item at lth location.

Then

$$a_{ijkl} = a_{ik} + a_{jl} - p_{ijkl}. \qquad \text{(2.2.1.4.1)}$$

where p_{ijkl} is the interaction factor as mentioned above. The expressions for the various costs mentioned above are given below. The notations used are similar to 2.2.1.2

$$a_{ik} = \sum_{o \in S} f_{io} * d_{k(lk,o)} \qquad \text{(2.2.1.4.2)}$$

$$a_{jl} = \sum_{o \in S} f_{jo} * d_{l(ll,o)} \qquad \text{(2.2.1.4.3)}$$

$$p_{ijkl} = \sum_{o \in S} b_{kl} * \text{Maximum}[f_{io} * d_{k,(lk,o)}, f_{jo} * d_{l,(ll,o)}] \qquad \text{(2.2.1.4.4)}$$

Where all the notations and the logic behind computation of this interaction term is same as that of the previous case.

The summation o, that is, over all the stored items, used in computing the various dissimilarity cost as mentioned above can be user specified also. The need for this arises from the fact that due to equipment limitations some locations may not be served by the same equipment. The other reason may be that due to space limitations some location may not be served in the same tour. Other possibility is that there may be some items which cannot be retrieved together. So based on these limitations a user can judge which items and locations are to be included in the computation of the dissimilarity cost.

2.3 Assumptions

In this section we will list the assumptions which we have made while formulating the problems in the next three chapters. The assumptions are :

[1]. Each bin will store only one type of item (part number) at a time.

[2]. Storage and retrieval requests are for the entire contents of the bin.

[3]. The warehouse under consideration, is a rectangular warehouse with an input dock and several output docks. There are several aisles parallel to each other. On each side of the aisle is a storage rack with R rows. The row space is continuous and the storage racks has vertical partitions.

[4]. Bin sizes are standardized and are multiple of unit size.

[5]. Several type of equipments are available, with constraints on the type of items(bins) they can carry and the locations which they can serve.

[6]. Interleaving is ignored. All storage and retrieval functions are initiated with the equipments at the input/output dock.

[7]. All the bins for an item are of same size.

[8]. The equipment acceleration and deacceleration time is ignored.

[9]. Store requests are served FCFS, that is, first come first serve basis.

2.4 Classification of the stock assignment problems

It will be convenient to have a simple notation to represent different types of stock assignment problems. We shall classify the problems according to five parameters : n / b / k / c / A.

n is the number of bins to be stored.

b is the type of items to be stored at the same time.

k is the number of equipment available for storing the items.

c describes equipment capacity, that is, whether the equipment can handle one bin at a time (c = 1) or it can handle more than one bin at a time.

A describes the retrieval pattern within the warehouse. A may be

- S for the case of single bin retrieval.
- M for the case of multiple bin retrieval.

As an example : n / 1 / 1 / c / M is the problem of locating "n" bins of same item, when no choice of equipment is available, the capacity of the equipments is "c" (that is more than one bin can be handled at a time) and retrieval pattern in the warehouse is multiple bins at a time.

CHAPTER III
DESCRIPTION , MATHEMATICAL MODELING AND SOLUTION METHODOLOGY FOR ONE BIN STORAGE

3.1 Description of the problem

In this chapter we shall consider a simple stock assignment problem, in which only one bin of single item is to be stored at a time. This problem can be further classified to subcases on the basis of two factors as follows :

(i) Order picking requirement (Type of retrieval)

There are two possible methods of retrieval of bins. Either the order are picked up by an equipment capable of handling one bin of a type (single bin retrieval) or are picked up by an equipment where more than one bin is picked up at the same time (multiple bin retrieval). The major impact on the stock assignment problem of, this factor is that in multiple retrieval, one will like to store the items, which are generally picked up together close to each other.

(ii) Choice of equipment

The warehouse may have only one type of equipment (single equipment case) for stock assignment or may have more than one type of equipment (multiple equipment case) for this purpose. In second case in addition to the location, choice of available equipment will also have to be taken into consideration, as the cost will be a function of location and equipment cost combination.

Thus on the basis of above two factors , there are four cases which will be discussed in this chapter :

[1] . To locate one bin of a single item, when choice of equipment is not available and retrieval of bins is one bin at a time.[1 / 1 / 1 / 1 / S]

[2] . To locate one bin of a single item ,when equipment choice is also available and retrieval of bins is one bin at a time.[1 / 1 / k / 1 / S]

[3]. To locate one bin of a single item, when choice of equipment is not available and retrieval of bins is multiple bins at a time.[1 / 1 / 1 / 1 / M]

[4]. To locate one bin of a single item ,when equipment choice is also available and retrieval of bins is multiple bins at a time.[1 / 1 / k / 1 / M]

The location of the bin will be selected so as to optimize the total cost which in this case will consist of :

(a) The storage cost :

We shall assume that the storage cost is a function of the distance to be traveled for the storage and the equipment selected. This will include a fixed cost for loading and unloading the equipment, start up cost and travel cost depending on the type of distance traveled (that is, horizontal, vertical and across the aisle movement).

(b) The retrieval cost :

The cost of retrieval will include the cost of picking up and taking the item upto the output dock. In case of multiple bin retrieval another cost component due to desirability of the item, which are likely to be retrieved together, to be stocked in close vicinity of each other is also to be taken into consideration. We shall call such a cost as cost of dissimilarity.

First we shall give the notations and variables that are used , after that the mathematical formulations and the solution methodologies for the problems shall be discussed.

3.2 Notations used

dI_i = Shortest horizontal distance of the ith location from the input port.

dO_i = Shortest horizontal distance of the ith location from the nearest output port.

dv_i = Vertical distance of the ith location from the ground level.

f = Frequency of retrieval of item to be stored. It is the ratio of the average number of bins of the item required in a specified time period to the total number of bins of all items retrieved in the same time period.

R = Set of locations which are preassigned to the item to be stored. If there is no preassignment then R is a set of all the locations in the warehouse.

A = Set of available (empty) locations in the warehouse.

S = Set of locations available to store the item(s) to be stored.

$$S = R \cap A$$

Also let $| S | = m$

w_1 = Weightage for the retrieval cost .

w_2 = Weightage for the dissimilarity cost .

Vh_e = Horizontal velocity of eth type of equipment.

Vv_e = Vertical velocity of the eth type of equipment.

Tl_e = Loading time of the eth type of equipment.

Tu_e = Unloading time of the eth type of equipment.

Oc_e = Operating cost of eth type of equipment.(in Rs. per unit time).

Lc = Labour cost (in Rs. per unit time).

E_i = Set of equipment's which can serve location i and can handle the item(bin) to be located.

3.3Variables used

In this chapter only two binary decision variables are used and, they are :

X_i =1 if location i is selected for locating the item.

= 0 otherwise

Z_{ie} = 1 if equipment e is selected for ith location.

= 0 otherwise.

3.4 Mathematical formulation and solution methodology

3.4.1 Case # 1 [1 / 1 / 1 / 1 / S]

Here we shall consider the problem with the following features:

Only one bin of an item is to be stored.

Retrieval of bins is one bin at a time.

Only one type of equipment is available (that is, no choice of equipment)

This is a simple problem. Only one bin of an item is to be located in one location out of available m locations. For a location storage cost and retrieval cost are function of the distance of that location from the input dock and the nearest output dock respectively. The distance function is same as that described in chapter II .

The storage cost of the location i with the only available equipment can be computed as :

$$ts_i = [{}^{dIi}/_{Vhe} + {}^{dvi}/_{Vve} + T_{le} + T_{ue}] * (L_c + O_{ce}) \ldots\ldots\ldots\ldots\ldots (3.4.1.1)$$

where the subscript 'e' denotes the only available equipment.

Similarly the retrieval cost associated with each location is computed as :

$$tr_i = [{}^{dOi}/_{Vhe} + {}^{dvi}/_{Vve} + T_{le} + T_{ue}] * (L_c + O_{ce}) \ldots\ldots\ldots\ldots\ldots (3.4.1.2)$$

where the subscript 'e' again denotes the only available equipment.

The cost associated with each location C_i can be computed as follows :

$$C_i = f\,(ts_i + w_1 * tr_i) \ldots\ldots\ldots\ldots\ldots (3.4.1.3)$$

The problem can be formulated as :

(P1)

$$\text{Minimize}\sum_{i\in S} C_i * X_i \ldots\ldots\ldots\ldots\ldots\ldots(3.4.1.4)$$

$$\text{S.t.}\ldots\ldots\ldots\sum_{i\in S} X_i = 1 \ldots\ldots\ldots\ldots\ldots\ldots(3.4.1.5)$$

$$\ldots\ldots\ldots\ldots X_i \in (0,1) \ldots\ldots\ldots\ldots\ldots\ldots(3.4.1.6)$$

Equation (3.4.1.4) is our objective function and depicts the total cost .

Constraint (3.4.1.5) ensures that only one location is selected out of the available m locations.

Constraint (3.4.1.6) is the integrality constraint.

The problem is a very simple one, and is equivalent to the following : given m numbers $C_1,\ldots\ldots\ldots\ldots C_m$ choose the smallest number. Thus the problem get solved by selecting the location with smallest cost C_i among all the locations.

3.4.2 Case # 2 [1 / 1 / k / 1 / S]

Here we shall consider the problem with the following features:

Only one bin of an item is to be stored.

Retrieval of bins is one bin at a time.

More than one type of equipment is available.

The problem is same as the problem of case # 1 except that best equipment is also to be selected for the selected location. As the choice of equipment is dependent only on the selected location, the problem can be solved in two steps.

In the first step we shall select for each of the available location, the optimal equipment out of the available equipments, and then in the second step we shall select the optimal location.

The problem is formulated as follows :

(P2)

$$\text{Minimize} \sum_{i \in S} \sum_{e \in Ei} ts_{ie} * Z_{ie} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4.2.1)$$

$$S.t. \ldots\ldots\ldots \sum_{e \in Ei} Z_{ie} = 1 \ldots\ldots \text{for.all.} i \in S \ldots (3.4.2.2)$$

$$\ldots\ldots\ldots\ldots\ldots Z_{ie} \in (0,1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4.2.3)$$

Equation (3.4.2.1) is our objective function and depicts the cost for serving each of the available location .

Constraint (3.4.2.2) ensures that only one equipment is selected for all the available locations.

Constraint (3.4.2.3) is the usual integrality constraint.

The problem can be solved by selecting the minimum cost equipment for each of the available locations. Let the solution be denoted by the set $\{ts_1^*, ts_2^*, \ldots\ldots ts_m^*\}$. In this way there will be a least cost equipment selected apriori for each of the available locations.

Now the problem is equivalent to the problem (P1) where the cost associated with each location, that is, C_i is computed as follows :

$$C_i = f\,(ts_i^* + w_1 * tr_i) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4.2.4)$$

where tr_i is the retrieval transportation cost associated with the location and is given by (3.4.1.2) . For retrieval, if there is choice of equipment then again tr_i is the minimum of all the possible tr_{ie}'s . The solution methodology in this case will be same as that adopted in solving the problem 3.4.1., that is, problem is equivalent to the following : given m numbers $C_1, \ldots\ldots\ldots\ldots C_m$ choose the smallest number. Thus the problem get solved by selecting the location with smallest cost C_i among all the locations. And for the selected location we will select the equipment selected apriori for it

3.4.3 Case # 3 [1 / 1 / 1 / 1 / M]

Here we shall consider the problem with the following features:

Only one bin of an item is to be stored.

More than one bin is retrieved in the same pick up

Only one type of equipment is available (that is, no choice of equipment)

Only one bin of an item is to be located in one location out of available m locations. But as retrieval of bins is more than one bin at a time, the similarity of items with other items also comes into the picture, it is desirable to store items which are likely to be retrieved together, close to each other. With each location a storage cost is associated which is proportional to the distance of that location from the input dock. There will be a retrieval cost associated with every location. As the retrieval of bins is multi bins at a time, this cost will be proportional to the distance of the location from the output dock, as well as to the distance of the location from the other locations where those items are stored which are generally retrieved with the item that is to be stored. We discussed this issue in the last chapter under the title of correlated assignment and quantified this cost as dissimilarity cost, that is a_i

Thus the cost associated with each location C_i can be computed as follows :

$$C_i = f\,(ts_i + w_1 * tr_i) + a_i \qquad (3.4.3.1)$$

Weight w_1 should account for the fact that retrieval cost is to be divided over a large number of items. For example w_1 can be taken as 1 / (average number of items in a single retrieval).

The problem formulation will remain same as that in the case 3.4.1. The objective function is same as (3.4.1.4) subjected to the constraints (3.4.1.5) and (3.4.1.6).

The solution methodology adopted in this case is also the same as adopted in case 3.4.1., that is, given m numbers C_1,............C_m choose the smallest number. Thus the problem get solved by selecting the location with smallest cost C_i among all the locations.

3.4.4 Case # 4 [1 / 1 / k / 1 / M]

Here we shall consider the problem with the following features.

Only one bin of an item is to be stored.

More than one bin is retrieved in the same pick up.

More than one type of equipment is available.

Here also the problem is same as the problem of 3.4.3 except that best equipment is also to be selected for the selected location. As the choice of equipment is dependent only on the selected location, the problem can be solved in two steps.

In the first step we shall select for each of the available location, the optimal equipment and then in the second step we shall select the optimal location.

So in the first step formulation and solution methodology is same as that adopted in 3.4.2. The objective function is same as (3.4.2.1) subjected to constraints (3.4.2.2) and (3.4.2.3). The solution to this problem will give us a minimum cost equipment for each of the available locations. Let the solution be denoted by the set $\{ts_1^*, ts_2^*, \ldots\ldots\ldots ts_m^*\}$. In this way there will be a least cost equipment selected apriori for each of the available locations.

In the second step of the problem we will formulate the problem as formulated in case 3.4.3 . The objective function and constraints will be same as that given by equations (3.4.1.4) - (3.4.1.6). Only the computation of cost associated with each location, that is, C_i will change and is computed as:

$$C_i = f\,(ts_i^* + w_1 * tr_i) + a_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.4.4.1)}$$

The solution procedure is same, that is,: given m numbers $C_1, \ldots\ldots\ldots\ldots C_m$ choose the smallest number. Thus the problem get solved by selecting the location with smallest cost C_i among all the locations and for the selected location select the equipment selected apriori for it. In case the best equipment is not available for the selected location then assign the second best equipment to that location and compute C_i^* for that location. Now update the list of C_i's by adding C_i^* to it and deleting C_i from it and select the location which have minimum value of C_i in this list. Keep on doing like this until a location with free best equipment is selected.

CHAPTER IV
DESCRIPTION , MATHEMATICAL MODELING AND SOLUTION METHODOLOGY FOR MULTI BIN STORAGE WHEN THE EQUIPMENT CAN HANDLE ONLY ONE BIN AT A TIME

4.1 Description of the problem

In this chapter we shall consider the problem where more than one bin either of the same item or of different items has to be stored. We shall assume that equipment / equipment(s) available for storage can carry only one bin at a time. This problem can be further classified on the following basis :

(i) Number of items to be stored

There are two possible cases, either all the bins are of one item or they are of different items. In the second case in addition to selection of location, assignment of the items to locations is also involved.

(ii) Order picking requirement (Type of retrieval)

This factor is same as that mentioned in the previous chapter, that is, the orders are picked up by an equipment capable of handling one bin of a type (single bin retrieval) or are picked up by an equipment where more than one bin is picked up at the same time (multiple bin retrieval). The major impact on the stock assignment problem of, this factor is that in multiple retrieval, one will like to store the items, which are generally picked up together close to each other, so the dissimilarity cost, that is, cost of not locating the item picked up at the same time, together is also to be considered.

(iii) Choice of equipment

This factor is also mentioned in the previous chapter. The warehouse may have only one type of equipment (single equipment case) for stock assignment or may have more than

one type of equipment (multiple equipment case) for this purpose. In second case in addition to the location, choice of available equipment will also have to be taken into consideration, as the cost will be a function of location and equipment cost combination.
On the basis of these three factors, following eight possible cases will be discussed in this chapter :

[1] . To locate n bins of same item, when choice of equipment is not available and retrieval of bins is one bin at a time.[n / 1 / 1 / 1 / S]

[2] . To locate n bins of same item, when equipment choice is also available and retrieval of bins is one bin at a time.[n / 1 / k / 1 / S]

[3]. To locate n bins of same item, when choice of equipment is not available and retrieval of bins is multiple bins at a time.[n / 1 / 1 / 1 / M]

[4]. To locate n bins of same item, when equipment choice is also available and retrieval of bins is multiple bins at a time.[n / 1 / k / 1 / S]

[5] . To locate n bins of different items, when choice of equipment is not available and retrieval of bins is one bin at a time.[n / b / 1 / 1 / S]

[6] . To locate n bins of different items, when equipment choice is also available and retrieval of bins is one bin at a time.[n / b / k / 1 / S]

[7]. To locate n bins of different items, when choice of equipment is not available and retrieval of bins is multiple bins at a time.[n / b / 1 / 1 / M]

[8].To locate n bins of different items, when equipment choice is also available and retrieval of bins is multiple bins at a time.[n / b / k / 1 / M]

The criteria for optimization remains the same as mentioned in the previous chapter, that is, minimization of the weighted storage and retrieval cost.

First we shall give the notations and variables that are used , after that the mathematical formulations and the solution methodologies for the problems shall be discussed.

4.2 Notations used

dI_i = Shortest horizontal distance of the ith location from the input port.

dO_i = Shortest horizontal distance of the ith location from the nearest output port.

dv_i = Vertical distance of the ith location from the ground level.

f_i = Frequency of retrieval of the ith item to be stored. It is the ratio of the average number of bins of the item required in a specified time period to the total number of bins of all items retrieved in the same time period.

R_i = Set of locations which are preassigned to the ith item to be stored.

A = Set of available (empty) locations in the warehouse.

S_i = Set of locations available to store the ith item to be stored.

$$S_i = R_i \cap A$$

also let $$| S_i | = m_i$$

w_1 = Weightage for the retrieval cost .

w_2 = Weightage for the dissimilarity cost .

Vh_e = Horizontal velocity of the eth type of equipment.

Vv_e = Vertical velocity of the eth type of equipment.

Tl_e = Loading time of the eth type of equipment.

Tu_e = Unloading time of the eth type of equipment.

Oc_e = Operating cost of eth type of equipment.(in Rs. per unit time).

Lc = Labour cost (in Rs. per unit time).

E_i = Set of equipment's which can serve location i.

F_i = Set of equipments which can serve item i .

L = Set of selected locations.

4.3 Variables used

In this chapter following binary variables are used :

(i) Same item storage case.

X_i = 1 if location i is selected for locating the item.

= 0 otherwise

Z_{ie} = 1 if equipment e is selected for location i

= 0 otherwise.

(ii) Different items storage case.

X_{ij} = 1 if item i is located at jth location.

= 0 otherwise.

Z_{ije} = 1 if item i is located at jth location by eth type of equipment.

= 0 otherwise.

4.4 Mathematical formulation and solution methodology

4.4.1 Case #1 [n / 1 / 1 / 1 / S]

In this case we consider the problem of :

Locating "n" bins of same item.

Retrieval of bins is one bin at a time.

Only one type of equipment is available (that is, no choice of equipment)

This is the simplest problem in this category with which we are dealing with. "n" bins of same item is to be located in "n" locations out of available "m" locations. With each location a storage cost is associated which is proportional to the distance of that location from the input dock and a retrieval cost which is proportional to the distance of the location from the nearest output dock. The distance function is same as that described in chapter II .

The storage cost of the location i with the only available equipment will be given by the expression (3.4.1.1) of section 3.4.1

Similarly the retrieval cost associated with each location will be given by equation (3.4.4.2) of section 3.4.1

Thus the cost associated with each location C_i can be computed as follows :

$$C_i = f\,(ts_i + w_1 * tr_i)\ldots\ldots\ldots\ldots\ldots\ldots(4.4.1.1)$$

The objective function will be same as that section 3.4.1. Only the constraint 3.4.1.5 will change . The formulation is given below :

(P3)

$$\text{Minimize}\sum_{i\in S} C_i * X_i \ldots\ldots\ldots\ldots\ldots\ldots(4.4.1.2)$$

$$\text{S.t.}\ldots\ldots\ldots \sum_{i\in S} X_i = n \ldots\ldots\ldots\ldots\ldots\ldots(4.4.1.3)$$

$$\ldots\ldots\ldots X_i \in (0,1) \ldots\ldots\ldots\ldots\ldots\ldots(4.4.1.4)$$

Equation (4.4.1.2) is our objective function and depicts the total cost .

Constraint (4.4.1.3) ensures that "n"locations are selected out of the available m locations.

Constraint (4.4.1.4) is the integrality constraint.

The problem is a very simple one, and is equivalent to the following : given m numbers $C_1,\ldots\ldots\ldots\ldots C_m$ choose those "n" numbers whose sum is the smallest . Thus the problem get solved by selecting "n" locations with the smallest sum of C_is among all the locations.

4.4.2 Case # 2 [n / 1 / k / 1 / S]

Here we shall consider the problem with the following features:

Locating "n" bins of same item.

Retrieval of bins is one bin at a time.

Choice of equipment is also there.

The problem is same as the problem of case # 1 of 4.4.1, except that best equipment is also to be selected for the selected locations. As the choice of equipment is dependent only on the selected location, the problem can be solved in two steps.

In the first step we shall select for each of the available location, the optimal equipment and then in the second step we shall select the optimal location.

So in the first step our objective is to minimize the storage cost ts_{ie}, the cost to store the item in the available location i with the eth equipment. Where ts_{ie} is given by equation (3.4.1.1). In this step our formulation will remain same as that in the case 3.4.2 . The objective function will be same as equation (3.4.2.1), subjected to the constraints (3.4.2.2) and (3.4.2.3) .

The solution to this problem will give us a minimum cost equipment for each of the available locations. Let the solution be denoted by the set $\{ts_1^*, ts_2^*, \ldots\ldots ts_m^*\}$. In this way there will be a least cost equipment selected apriori for each of the available locations, provided that there are unlimited number of each type of equipment. In case the number of equipments are limited, the problem will get formulated as a generalized assignment problem, which is a special case of transportation problem. This problem and its solution methodology is described in section 4.4.6.

In the second step of the problem we will formulate the problem as formulated in case 4.4.1. The objective function and constraints will be same as that given by equations (4.4.1.2) - (4.4.1.4). Only the computation of cost associated with each location, that is, C_i will change and is computed as:

$$C_i = f\,(ts_i^* + w_1 * tr_i) \ldots\ldots\ldots\ldots (4.4.2.1)$$

where tr_i is the retrieval transportation cost associated with the location and is given by (3.4.1.2) . For retrieval, if there is choice of equipment then again tr_i is the minimum of all the possible tr_{ie}'s . The solution methodology in this case will be same as that adopted in solving the problem 3.4.1.,that is, problem is equivalent to the following : given m numbers $C_1, \ldots\ldots\ldots C_m$ choose those "n" numbers whose sum is the smallest. And for the selected locations we will select the equipment selected apriori for it. In this case also if the best equipment is not available for the selected locations then assign the second best equipment to that location and compute C_i^* for that location. Now update the list of C_i's

by adding C_i^* to it and deleting C_i from it . The equipment availability can be computed by multiplying the number of equipment of that type with the number of times that equipment can be used in the time to be taken for storing all the items.

4.4.3 Case # 3 [n / 1 / 1 / 1 / M]

In this case we consider problem of :

Locating "n" bins of same item.

Retrieval of bins is more than one bin at a time.(Multiple retrieval)

Only one type of equipment is available (that is, no choice of equipment)

In this case the cost function will be more complex as compared to the previous cases. While the storage cost associated with each location can be computed as in the previous cases, the retrieval cost will be a function of both the distance from the output dock and the relative position from those items, which are generally picked up at the same time with the item under consideration. We discussed this issue in the second chapter under the title of correlated assignment and quantified the cost as dissimilarity cost, that is, a_i

Therefore the storage cost associated with every location = $f(ts_i)$............(4.4.3.1)

The retrieval cost associated with each location = $f(tr_i) + a_i$......................(4.4.3.2)

Therefore the cost associated with every location, C_i is given by:

$$C_i = f(ts_i + w_1 tr_i) + w_2 * a_i \text{..} (4.4.3.3)$$

As the items which are to be located have interaction between them so there will be an interaction cost also if the item is stored both at locations i and j and is given by C_{ij}. These interaction term is also discussed in the second chapter and is given by

$$C_{ij} = w_2 * p_{ij} \text{..} (4.4.3.4)$$

So the problem can be formulated as :

(P4)

$$\text{Minimize}[\sum_{i\in S} C_i * X_i - \sum_{i\in S}\sum_{j\in S} C_{ij} * X_i * X_j] \quad (4.4.3.5)$$

$$\text{S.t.} \quad \sum_{i\in S} Xi = n \quad (4.4.3.6)$$

$$X_i \in (0,1) \quad (4.4.3.7)$$

The objective function is different from the previous case but both the constraints are similar to the previous cases.

The problem formulated above is closely associated with the Weighted Maximum Clique Problem and the Time Tabling problem. Our problem can be reduced to a graph G having m nodes corresponding to each available locations. With each node is associated a cost C_i. Each arc (i,j) corresponds to the cost associated with the dissimilarity of storing the item both at locations i and j, that is, C_{ij}

So in the network environment our problem is to find a complete subgraph of G (whose vertices are pairwise adjacent) containing n nodes and the total cost, that is, cost associated with the nodes and the arcs in the subgraph is to be minimized.

4.4.3.1 Maximum Weight Clique Problem

The maximum-weight clique problem, or MWCP for short, goes as follows : given a graph whose vertices and edges carry numerical weights, find a clique (that is a set of pairwise adjacent vertices) whose total weight is as large as possible. This problem is notoriously hard, and the problem of deciding whether a prescribed graph contains a clique of prescribed size is NP-Complete. [Balas, Chvatal and Jaroslav[3]]

The analogy of our problem to MWCP is that we have to find a clique of size n from a graph of size m such that the sum of total weights associated with the edges and vertices in the clique is minimized. As MWCP is a maximization problem we can convert our problem to maximization problem by negating the weights associated with the edges and the vertices of the graph.

4.4.3.2 Time Tabling Problem

Similarly the general university timetabling problem involves assigning professors to courses and courses to time slots and classrooms. In the weighted graph model for a timetabling problem, the vertices represent the courses to be scheduled and the weights associated with the edges represent the varying severity of conflicts between courses. There are at most k time slots that can be used for the timetable. With each vertex is associated a cost vector having k non negative components. The ith component represents the cost incurred when the vertex is assigned the ith time slot (that is colored with the ith color in reference to the graph coloring problem). The objective is to find a least cost k-coloring of the graph.[Lynn Kiaer and Jay Yellen[26]]

Our problem is a specific case of time tabling problem where k=2 (that is, the time slots available), that is, only two cases are possible, either the node is selected or it is not. And the cost associated with the vertices is also two dimensional, the cost incurred if the vertex is selected is equal to C_i otherwise it is equal to zero.

As both the problems, that is, MWCP and Time Tabling Problem are NP-Complete, so the only way to solve these problems in reasonable amount of time is to develop some heuristics that give approximate solution which is not far away from the optimal solution. We have developed some heuristics for our problem based on available heuristics for MWCP. Exact algorithms to MWCP are also known, but as our application requires on-line computing, exact algorithms are unlikely to be useful.

4.4.3.3 Heuristic # 1 [SLP Heuristic]

The simplest possible heuristic is to neglect the interaction cost (that is, C_{ij}) between the items to be located and treat this problem as of locating n single bins sequentially. It may be noted that each time a bin is located, for all subsequent available locations the interaction cost a_i, has to be recomputed. The steps involved will be as follows:

Step I Initialize

count = 0 {The number of bins that has been located so far}

S = {1,2,..........m} { The set of all the available locations}

O = {1,2,..........o} { The set of all the locations already filled in the warehouse}

Step II Update a_i and compute C_i for all $i \in S$

Step III Choose the location (i) with the least value of C_i

Step IV Update

$S = S - \{i\}$

$O = O + \{i\}$

count = count + 1

Step V If count = n then go to Step VI , else go to Step II.

Step VI Stop.

4.4.3.4 Heuristic # 2 (HTT Heuristic)

Our next heuristic is based on a heuristic algorithm for university course time tabling suggested by Lynn Kiaer and Jay Yellen[26].

As we have already described the university course time tabling problem, its weighted graph model and its analogy with the problem formulated in this section, we straight away will describe the terms used in the heuristic algorithm for the time tabling problem and the algorithm for our problem. This heuristic selects nodes sequentially. However it may be noted that in our problem the node weight and edge weight will change each time a node is selected, unlike the time table problem, and hence these values have to be updated after selection of each node.

cdeg(v) $\rightarrow$Cost degree of a vertex v is defined as the weight associated with the node v. With reference to our problem this is C_v.

udeg(v)$\longrightarrow$ Uncolored (weighted) degree of a vertex v is defined as the sum of the weights of those edges that join v to those vertices that have not yet been traversed, that is neither selected nor rejected. With reference to our problem this is

$$\sum_{\substack{j \in S \\ j \neq v}} C_{vj}$$

tdeg(v)$\longrightarrow$ Total (weighted) degree of v. With reference to our problem

$$tdeg(v) = cdeg(v) - udeg(v)$$

$$tdeg(v) = C_v - \sum_{\substack{j \in S \\ j \neq v}} C_{vj}$$

Step I Initialize

count = 0 {The number of bins that has been located so far}

S = {1,2,.. ....m} { The set of all the available locations}

O = {1,2,...........o} { The set of all the locations already filled in the warehouse}

Step II Recompute a_i, p_{ij}, and tdeg(v) for all $v \in S$

Step III Choose the location (v) with the least value of tdeg

Step IV Update

S = S - {i}

O = O + {i}

count = count + 1

Step V If count = n then go to Step VI , else go to Step II.

Step VI Stop.

4.4.3.5 Heuristic # 3 [HIP Heuristic]

The two heuristics which we discussed in the previous sections are constructive heuristics. A further improvement in the results obtained from these heuristics can be obtained by pairwise exchanging the locations where the items are located. So the next heuristic which we are going to present is an improvement heuristic, based on pairwise exchange heuristics, such as CRAFT for plant layout.

Suppose

L $\rightarrow$ {Set of selected locations from heuristic 1 or 2)

$|L| = n$

NA $\rightarrow$ {Set of now available locations}

$|NA| = m-n$

Totcost(L) $\rightarrow$ Total cost of locating n items at n locations in set L.

So we can compute the impact of changing the locations from L to NA and vice versa and if there is a reduction in the total cost, we can switch the locations.

DTCij(L) $\rightarrow$ Change in the total cost when location i in set L is interchanged with the location j in set NA.

e $\rightarrow$ It is the maximum of zero and the greatest decrease in Totcost(L) found so far for the given assignment.

This heuristic can be described as follows :

Step I Compute Totcost(L)

Step II Set e = 0

Step III Set i = 1 ($i \in L$)

Step IV Set j = 1 ($j \in NA$)

Step V Calculate DTCij(L)

Step VI If DTCij(L) is greater than e, go to Step VII else go to Step VIII

Step VII e = DTCij(L)

Step VIII If $j = m-n$, go to Step IX else go to Step X

Step IX If $i = n$, go to Step XII, else go to Step XI

Step X Increment j by 1 and go to Step V

Step XI Increment i by 1 and go to Step IV

Step XII If e is positive, go to Step XIII else go to Step XV

Step XIII Replace Totcost(L) by Totcost(L) - e

Step XIV Revise the assignment L by interchanging the locations i and j and recompute a_i p_{ij} and go to Step II

Step XV Stop.

The accuracy of this heuristic can further be improved by interchanging two or three locations at a time instead of one.

4.4.3.6 Exact algorithm

There are some exact algorithms also for the solution of the maximum clique problem. Pardalos and Rodgers [34] gave a branch and bound algorithm for the maximum clique problem. They gave an exact algorithm which works on the principle of branch and bound method but computational time is reduced through an initial ordering of vertices.

Likewise there are some other exact algorithms but the problem with all of them is that the required computing time grows exponentially with the size of the problem. As the stock assignment problem is to be solved on line, exact algorithms are unlikely to be useful for our case.

4.4.4 Case # 4 [n / 1 / k / 1 / M]

Here we shall consider the problem with the following features:

Locating "n" bins of same item.

Retrieval of bins is multiple bins at a time.

Choice of equipment is also there.

The problem is same as the problem of case # 1 of 4.4.3, except that best equipment is also to be selected for the selected locations. As the choice of equipment is dependent only on the selected location, the problem can be solved in two steps as in case 4.4.2 . In the first step we shall select for each of the available location, the optimal equipment and then in the second step we shall select the optimal location.

So in the first step our objective is to minimize the storage cost ts_{ie}, the cost to store the item in the available location i with the eth equipment. Where ts_{ie} is given by equation (3.4.1.1) .In this step our formulation will remain same as that in the case 3.4.2 . The objective function will be same as equation (3.4.2.1), subjected to the constraints (3.4.2.2) and (3.4.2.3) .

The solution to this problem will give us a minimum cost equipment for each of the available locations. Let the solution be denoted by the set $\{ts_1^*, ts_2^*, \ldots\ldots ts_m^*\}$. In this way there will be a least cost equipment selected apriori for each of the available locations, assuming that unlimited equipment of a type are available or alternatively the turnaround time for the equipment is small so that the same equipment can be reassigned as often as needed.

In the second step of the problem we will formulate the problem as formulated in case 4.4.3 . The objective function and constraints will be same as that given by equations (4.4.3.5) - (4.4.3.7). Only the computation of cost associated with each location, that is, C_i will change and is computed as:

$$C_i = f\,(ts_i^* + w_1 {*} tr_i) + w_2 {*} a_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.4.4.1)$$

where tr_i is the retrieval transportation cost associated with the location and is given by (3.4.1.2) . For retrieval, if there is choice of equipment then again tr_i is the minimum of all the possible tr_{ie}'s . The solution methodology in this case will be to select "n" locations from the heuristics for case 4.4.3 and for the selected locations to select the equipment selected apriori for it. In case when the number of equipments for each type are limited, locations will be selected sequentially from this list, and in case at a particular choice,

equipment is not available then C_i will be recomputed with respect to unavailable equipment and list resorted.

4.4.5 Case # 5 [n / b / 1 / 1 / S]

In this case the problem discussed has the following features :

To locate n bins of b different type of items .(Multi item case)

Retrieval of bins is one bin at a time.

Only one type of equipment is available (No choice of equipment is there).

This is the case when we have to locate n bins of b different type of items . The number of bins are n_1, n_2,...................n_b of b items respectively Here the cost function will comprise of two terms, a storage cost term which is proportional to the distance of the location from the input dock and a retrieval cost term which is again proportional to the distance of the location from the nearest output dock.

The cost of locating ith type of item at jth location C_{ij} is given by :

$$C_{ij} = f_i * [ts_j + w_1 * tr_j] \quad\text{...(4.4.5.1)}$$

The problem can be formulated as :

(P5)

$$\text{Minimize} \sum_{i=1}^{i=b} \sum_{j \in S_i} C_{ij} * X_{ij} \quad\text{..................................(4.4.5.2)}$$

$$\text{S.t.}\text{..........} \sum_{j \in S_i} X_{ij} = n_i \text{.......for..} i = 1,2,\text{..........} b \text{..(4.4.5.3)}$$

$$\text{.................} \sum_{i=1}^{i=b} X_{ij} \le 1 \text{........for..} j = 1,2,\text{..........} m \text{.(4.4.5.4)}$$

$$\text{.......... ....} X_{ij} \in (0,1) \text{.....................................(4.4.5.5)}$$

Equation 4.4.5.2 is the objective function and depicts the total cost.

Constraint 4.4.5.3 ensures that for each item type the number of locations selected must be equal to the number of bins of that item.

Constraint 4.4.5.4 ensures that no two items are assigned to the same location.

Constraint 4.4.5.5 is the integrality constraint.

The above problem is a special case of transportation problem and is known as Generalized Assignment Model. In fact, the common way to solve the generalized assignment problem is as a transportation problem.

4.4.5.1 Transportation problem

The transportation problem which has integer values on the right hand side of all the constraints has integer solutions property, that is, all the basic variables(allocation) in every basic feasible solution (including an optimal one) also have integer values. So it is a special type of linear programming problem that can be solved by applying the simplex method. However more efficient algorithms are known to this problem which exploit the special structure of the problem. Methods for solving the transportation problem are presented in most introductory operations research text, such as Hillier & Lieberman[19]. The procedure adopted to solve the transportation problem is that, first an initial basic feasible solution is obtained by one of the available criterion and then the solution is checked for dual feasibility. If the solution is dual infeasible, that is, the solution is improved by change of the primal solution. While efficient algorithms are known for this problem, we shall use some simple heuristics to give us a good solution as we are solving the problem on line. Simplest heuristic in this case will be to use Vogel's Approximation Method or Russel's Approximation Method . These are the methods used for obtaining the initial basic feasible solution for applying u-v method, but the solution obtained from these methods are not far away from optimal. In the next section we are going to describe these

two heuristics modified to suit our problem. The modification is required due to change in value of dissimilarity cost, once a selection is made.

4.4.5.2 Heuristic # 1 [HVOGEL]

This heuristic is based on Vogel's approximation method.[Hillier & Lieberman[19]].

Step I Initialize

$I = \{1,2,............b\}$ { Set of items to be stored}

$A=\{1,2,............m\}$ { Set of available locations}

$S_i=\{1,2,...........mi\}$ { Set of available locations which can store ith item}

$count[i] = 0$ for $i \in I$ { The number of bins of item i stored so far}

Step II Calculate

$C_{ij} = f_i * [ts_j + w_1 * tr_j]$ for $i \in I, j \in S_i$

$C_{ij} =$ infinity for $i \in I, j \notin S_i, j \in A$

Step III For each item $i \in I$ calculate

difference[i] = {minimum C_{ij} - next minimum C_{ij} } over all j

Step IV For each location $j \in A$ calculate

difference[j] = {minimum C_{ij} - next minimum C_{ij} } over all i

Step V Find out the minimum of all the differences calculated in Step III and Step IV

Step VI If minimum calculated in Step V is a difference from Step III then

assign item i of the minimum difference[i] to the least cost location available. Else the location j of the minimum difference[j] is assigned to the minimum cost item

Step VII Update

A = A - {selected location(j)}

count[located item(i)] = count[located item(i)] + 1

If count[i] = n_i then

I = I - {i}

Step VIII If I = { } then go to Step IX else go to Step II

Step IX Stop.

4.4.5.3 Heuristic # 2 [HRUSSEL]

This heuristic is based on Russell's approximation method.[Hillier & Lieberman[19]].

Step I Initialize

$I = \{1,2,............b\}$ { Set of items to be stored}

$A=\{1,2,............m\}$ { Set of available locations}

$S_i=\{1,2,...........mi\}$ { Set of available locations which can store ith item}

count[i] = 0 for $i \in I$ { The number of bins of item i stored so far}

Step II Calculate

$C_{ij} = f_i * [ts_j + w_1 * tr_j]$ for $i \in I, j \in S_i$

C_{ij} = infinity for $i \in I, j \notin S_i, j \in A$

Step III For each item $i \in I$ and each location $j \in A$ calculate

$D_{ij} = C_{ij}$ - MaximumC_{ik}(over all $k \in A$) - MaximumC_{kj}(over all $k \in I$)

Step IV Select the D_{ij} having the largest (in absolute terms) negative value. Assign the item (i) to location (j) corresponding to this D_{ij}

Step V Update

A = A - {selected location(j)}

count[located item(i)] = count[located item(i)] + 1

If count[i] = n_i then

I = I - {i}

Step VIII If I = { } then go to Step IX else go to Step II

Step IX Stop.

4.4.6 Case # 6 [n / b / k / 1 / S]

In this case the problem discussed has following features :

To locate n bins of b different type of items .

Retrieval of bins is one bin at a time.

Choice of equipment is also there.

The problem is same as the problem of case # 5 of 4.4.5, except that best equipment is also to be selected for the selected locations. In this case we cannot adopt the same strategy as adopted for the previous cases when choice of equipment was also there. The reason is that, in this case the bins are of different type and hence equipment choice will be a function of the item to be stored (i.e, bin type) and the location of that storage. So, in this case the location selection and equipment selection has to be formulated in a single stage problem.

The cost computation is some what similar to the previous case. The cost of storing the ith type of item at jth location on eth type of equipment is given by :

$$C_{ije} = f_i * [ts_{je} + w_1 * tr_j] \qquad (4.4.6.1)$$

where ts_{je} is given by (3.4.1.1) and tr_j is the retrieval transportation cost associated with the location, given by (3.4.1.2) . For retrieval, if there is choice of equipment then again tr_j is the minimum of all the possible tr_{je}'s .

The problem can be formulated as :

(P6)

$$\text{Minimize} \sum_{i=1}^{i=b} \sum_{j \in S_i} \sum_{e \in F_i \cap E_j} C_{ije} Z_{ije} \qquad (4.4.6.2)$$

$$\text{S.t.} \sum_{i=1}^{i=b} \sum_{e \in F_i \cap E_j} Z_{ije} \leq 1 \quad \text{for } j \in A \qquad (4.4.6.3)$$

$$\sum_{j \in S_i} \sum_{e \in F_i \cap E_j} Z_{ije} = n_i \quad \text{for } i = 1, 2, \ldots, b \qquad (4.4.6.4)$$

$$\sum_{i=1}^{i=b} \sum_{j \in S_i} Z_{ije} \leq p_e \quad \text{for all } e \qquad (4.4.6.5)$$

$$Z_{ije} \in (0,1) \qquad (4.4.6.6)$$

Where p_e is the number of equipment of type e multiplied with the number of times that equipment can be used in the estimated time taken for storing all the bins.

Equation 4.4.6.2 is the objective function.

Constraint 4.4.6.3 ensures that no location is assigned to more than one item or equipment.

Constraint 4.4.6.4 ensures that exactly n_i numbers of locations and equipments are selected for the ith type of item.

Constraint 4.4.6.5 ensures that the number of equipments assigned should not exceed the availability.

Constraint 4.4.6.6 is the usual integrality constraint.

This is essentially a three dimensional transportation problem. The heuristic suggested for the transportation problem can be extended to this case also.We will extend the heuristic 4.4.5.2 for this case.

4.4.6.1 Heuristic # 1 [HGVOGEL]

This heuristic is based on Vogel's approximation method .[Hillier & Lieberman[19]]. The best available equipment at the time of the cell allocation is assigned to the cell and problem is then reduced to a sequential transportation problem.

Step I Initialize

$I = \{1,2,............b\}$ { Set of items to be stored}

$A=\{1,2,............m\}$ { Set of available locations}

$S_i=\{1,2,...........mi\}$ { Set of available locations which can store ith item}

count[i] = 0 for $i \in I$ { The number of bins of item i stored so far}

Step II Calculate

C_{ij} =Minimum{ $f_i * [ts_{je} + w_1 * tr_j]$ }over all $e \in Ej \cap Fi$ for $i \in I, j \in S_i$

C_{ij} = infinity for $i \in I, j \notin S_i, j \in A$

Step III For each item $i \in I$ calculate

difference[i] = {minimum C_{ij} - next minimum C_{ij} } over all j

Step IV For each location $j \in A$ calculate

difference[j] = {minimum C_{ij} - next minimum C_{ij} } over all i

Step V Find out the minimum of all the differences calculated in Step III and Step IV

Step VI If minimum calculated in Step V is a difference from Step III then

assign item i of the minimum difference[i] to the least cost location available. Else the location j of the minimum difference[j] is assigned to the minimum cost item

Step VII Update

The equipment availability list.

A = A - {selected location(j)}

count[located item(i)] = count[located item(i)] + 1

If count[i] = n_i then

I = I - {i}

Step VIII If I = { } then go to Step IX else go to Step II

Step IX Stop.

4.4.7 Case # 7 [n / b / 1 / 1 / M]

The problem in this case is :

To locate n bins of b different type of items .

Retrieval of bins is multiple bins at a time.

Only one type of equipment is available (Equipment choice is not there).

In this case the cost function will be more complex as compared to the previous cases. While the storage cost associated with each location can be computed as in the previous cases, the retrieval cost will be a function of both the distance from the output dock and the relative position from those items, which are generally picked up at the same time with the item under consideration. We discussed this issue in the second chapter under the title of correlated assignment and quantified the cost as dissimilarity cost, that is, a_{ik}

Therefore the storage cost for ith type of item at kth location = $f_i\,(ts_k)$.............(4.4.7.1)

The retrieval cost for the ith type of item from kth location = $f_i\,(tr_k).+a_{ik}$.......(4.4.7.2)

Therefore the cost for locating ith type of item at kth location,that is, C_{ik} is given by:

$$C_{ik} = f_i(ts_k + w_1 * tr_k) + w_2 * a_{ik} \qquad (4.4.7.3)$$

As the items which are to be located have interaction between them so there will be an interaction cost also if the item is stored both at locations i and j and is given by C_{ijkl}. These interaction term is also discussed in the previous chapter and is given by

$$C_{ijkl} = w_2 * p_{ijkl} \qquad (4.4.7.4)$$

The problem can be formulated as :

(P7)

$$\text{Minimize} \sum_{i=1}^{i=b} \sum_{k \in S_i} C_{ik} * X_{ik} - \sum_{i=1}^{i=b} \sum_{j=1}^{j=b} \sum_{k \in S_i} \sum_{l \in S_j} C_{ijkl} * X_{ik} X_{jl} \qquad (4.4.7.5)$$

$$\text{S.t.} \ldots \sum_{j \in S_i} X_{ik} = n_i \ldots \text{for..} i = 1,2,\ldots b \qquad (4.4.7.6)$$

$$\ldots \sum_{i=1}^{i=b} X_{ik} \le 1 \ldots \text{for..} k = 1,2,\ldots m \qquad (4.4.7.7)$$

$$\ldots X_{ik} \ldots \in (0,1) \qquad (4.4.7.8)$$

The problem formulated above belongs to a special class of problem known as Generalized Quadratic Assignment Problem (GQAP).

4.4.7.1 Quadratic Assignment Problem

The QAP is, computationally, one of the most difficult combinatorial optimization problems.QAP as discussed in the literature is NP-hard [E.L. Lawler[28]]. Procedures that have been developed to attempt to find least cost solutions to the QAP are of two type : exact and heuristics. Exact procedures do in fact find least cost assignments. With the exception of total enumeration, all the exact procedures of interest developed thus far are

branch and bound, or implicit enumeration procedures. Because exact solution procedures have not yet been developed to the point where they are of significant practical value, heuristic procedures have received considerable attention. Many heuristic methods have been developed to solve the QAP [4],[41],[42] ,[44].

The heuristic methods can be classified in two groups :

(i) Constructive procedures.

(ii) Improvement procedures.

In general a constructive procedure can be looked upon as an n-stage decision process in which at the k-th stage, an element belonging to I(items) has to be assigned to S(locations). Improvements procedures takes an initial assignment as an input and then operate on it to give an improved solution. These two procedures, that is, constructive and improvement are generally used in conjuction. We suggest one heuristic from both the type.

4.4.7.2 Heuristic # 1 [HQVOGEL]

The simplest heuristic in this case will be to neglect the quadratic term in the objective function and solve the problem as a transportation problem. The Heuristics suggested in section 4.4.5.2 or 4.4.5.3 can be used to solve the transportation problem. Only the cost computation, that is, the computation of C_{ik}'s will be slightly different for this problem In this section we will quote the heuristic from section 4.4.5.2 with the changed cost computation formula.

Step I Initialize

$I = \{1,2,............b\}$ { Set of items to be stored}

$A=\{1,2,............m\}$ { Set of available locations}

$S_i=\{1,2,...........mi\}$ { Set of available locations which can store ith item}

count[i] = 0 for $i \in I$ { The number of bins of item i stored so far}

Step II Calculate

$C_{ik} = f_i(ts_k + w_1 * tr_k) + w_2 * a_{ik}$ for $i \in I$, $k \in S_i$

C_{ik}= infinity for $i \in I$, $k \notin S_i$, $k \in A$

Step III For each item $i \in I$ calculate

difference[i] = {minimum C_{ik} - next minimum C_{ik} } over all k

Step IV For each location $k \in A$ calculate

difference[k] = {minimum C_{ik} - next minimum C_{ik} } over all i

Step V Find out the minimum of all the differences calculated in Step III and Step IV

Step VI If minimum calculated in Step V is a difference from Step III then

assign item i of the minimum difference[i] to the least cost location available. Else

the location kof the minimum difference[k] is assigned to the minimum cost item

Step VII Update

A = A - {selected location(k)}

count[located item(i)] = count[located item(i)] + 1

If count[i] = n_i then

I = I - {i}

Step VIII If I = { } then go to Step IX else go to Step II

Step IX Stop.

Similarly the heuristic suggested in 4.4.5 3, based on Russel's approximation method can be used with the changed cost computation as in Step II of the above heuristic.

4.4.7.3 Heuristic # 2 [HQIMP]

To improve the results obtained by the constructive heuristics we suggest an improvement procedure known as steepest-descent pairwise-interchange procedure, which forms the basis of CRAFT, a computerized layout program. The procedure may be characterized as a steepest-descent procedure because it makes that interchange which results in the greatest decrease in the total cost.[Francis and White,[10]].

Before presenting the heuristic in algorithmic form we will define some terms used in the heuristic:

B⟶ An assignment obtained from the constructive heuristic

Totcost(B)⟶Total cost of assignment B given by equation (1) of section 4.4.7

DTCij(B)⟶Change in the total cost when location of item i is interchanged with the location of item j in the assignment B. Empty locations are considered to contain a dummy item.

e⟶It is the maximum of zero and the greatest decrease in Totcost(B) found so far for the given assignment.

The heuristic can be described as follows :

Step 0 Given : B

Step I Compute Totcost(B)

Step II Set e = 0

Step III Set i = 1 and j = 2

Step IV Calculate DTCij(B)

Step V If DTCij(B) is greater than e, go to Step VI else go to Step VII

Step VI Set e = DTCij(B) , set u to i , and set v to j

Step VII If j = m , go to Step VIII else go to Step IX

Step VIII If i = m-1 , go to Step XI, else go to Step X

Step IX Increment j by 1 and go to Step IV

Step X Increment i by 1, let j = i+1 , and go to Step IV

Step XI If e is positive, go to Step XII else go to Step XIV

Step XII Replace Totcost(B) by Totcost(B) - e

Step XIII Revise the assignment B by interchanging the locations of items i and j and recompute the values of a_{ij}'s, p_{ijkl}'s and go to Step II

Step XIV Stop.

4.4.7.4 GRASP heuristic for QAP

A Greedy Randomized Adaptive Search Procedure (GRASP) is a randomized heuristic that has been shown to quickly produce good quality solutions for a wide variety of combinatorial optimization problems. Li, Pardalos and Resende[30], in their paper have described a GRASP for the quadratic assignment problem.

The basic concepts of GRASP are construction and local search algorithms. When applied to QAP, the permutations are formed in two phases : (i) the first two assignments, and (ii) the remaining n-2 assignments. The first two assignments are first made simultaneously in the first construction iteration, while the remaining n-2 are made one assignment per construction iteration. For the initial two assignments, the greedy choice is the pair of assignments with the with the minimum cost of interaction.(As per the notations used in our context it means the assignments with the minimum value of $C_{ik}+C_{jl}-C_{ijkl}$). The final n-2 assignments use as the greedy choice the assignment that has the minimum cost interaction with the already made assignments. The local search implemented in this GRASP is a two exchange heuristic. For details, the interested readers shall refer [30].

4.4.7.5Exact methods for QAP

As it has already mentioned that with the exception of total enumeration, all the exact procedures of interest developed thus far are branch and bound, or implicit enumeration procedures. These procedures becomes computationally infeasible if n, the number of items to be located are more than 15 or 20. For any problem that can be solved by branch and bound, a number of different branch and bound procedures are conceivable. The amount of literature on the subject is large, and you are referred to references such as Gavett and Plyter[12], Gilmore[13], Hanan and Kurtzberg[16], Lawler[29] and Pierce and Crowston[36].

4.4.8 Case # 8 [n / b / k / 1 / M]

In this case the problem discussed has following features :

To locate n bins of b different type of items .

Retrieval of bins is multiple bins at a time.

Choice of equipment is also there.

The problem is same as the problem of case # 7 of 4.4.7, except that best equipment is also to be selected for the selected locations. In this case the strategy adopted is same as that adopted in Case # 6 of 4.4.6, that is, location and equipment selection is done simultaneously. The reason is same that, in this case the bins are of different type and we don't know beforehand that which bin will be stored in which location and so what all are the equipments which can serve that bin as well as that location. The cost computation is some what similar to the previous case. The cost of storing the ith type of item at kth location on eth type of equipment is given by :

$$C_{ike} = f_i * [ts_{ke} + w_1 * tr_k] + a_{ik} \text{...} (4.4\ 8.1)$$

where ts_{ke} is given by (3.4.1.1) and tr_k is the retrieval transportation cost associated with the location, given by (3.4.1.2) . For retrieval, if there is choice of equipment then again tr_k is the minimum of all the possible tr_{ke}'s .

The problem can be formulated as

$$\text{Minimize} \sum_{i=1}^{i=b} \sum_{k \in Si} \sum_{e \in Fi \cap Ej} C_{ike} * Z_{ike} - \sum_{i=1}^{i=b} \sum_{j=1}^{j=b} \sum_{k \in Si} \sum_{l \in Sj} C_{ijkl} * X_{ik} X_{jl} \text{..........} (4.4.8.2)$$

subjected to the constraints (4.4.6.2)- (4.4.6.6) and (4.4.7.6)-(4.4.7.8).

The simplest heuristic in this case will be to neglect the quadratic term in the objective function and apply the heuristic 4.4.7.2

4.4.8.1 Heuristic # 1

Step I   Initialize

$I = \{1,2,\ldots\ldots\ldots\ldots b\}$ { Set of items to be stored}

$A=\{1,2,\ldots\ldots\ldots\ldots m\}$ { Set of available locations}

$S_i=\{1,2,\ldots\ldots\ldots\ldots mi\}$ { Set of available locations which can store ith item}.

count[i] = 0 for $i \in I$ { The number of bins of item i stored so far}

Step II Calculate

C_{ik} = Minimum $\{f_i(ts_{ke}+w_1*tr_k)+w_2*a_{ik}\}$ over all $e \in Ej \cap Fi$ for $i \in I$, $k \in S_i$

C_{ik}= infinity for $i \in I$, $k \notin S_i$, $k \in A$

Step III For each item $i \in I$ calculate

difference[i] = {minimum C_{ik} - next minimum C_{ik} } over all k

Step IV For each location $k \in A$ calculate

difference[k] = {minimum C_{ik} - next minimum C_{ik} } over all i

Step V Find out the minimum of all the differences calculated in Step III and Step IV

Step VI If minimum calculated in Step V is a difference from Step III then

assign item i of the minimum difference[i] to the least cost location available. Else the location k of the minimum difference[k] is assigned to the minimum cost item

Step VII Update

The equipment availability.

A = A - {selected location(j)}

count[located item(i)] = count[located item(i)] + 1

If count[i] = ni then

I = I - {i}

Step VIII If I = { } then go to Step IX else go to Step II

Step IX Stop.

CHAPTER V

DESCRIPTION , MATHEMATICAL MODELING AND SOLUTION METHODOLOGY FOR MULTI BIN STORAGE WHEN THE EQUIPMENT CAN HANDLE MORE THAN ONE BIN AT A TIME

5.1 Description of the problem

In this chapter we shall consider the most general case of bin storage, that is, n bins are to be stored and available equipment's can handle more than one bin at a time. This case is further extended to subcases on the basis of three factors :

(i) Number of items to be stored

(ii) Type of retrieval

(iii) Choice of equipment

The reasons for classification on basis of these three factors are same as mentioned in the fourth chapter. So on the basis of these three factors the same eight cases will be discussed in this chapter.

5.2 Problem description and solution methodology

5.2.1 Case # 1 [n / 1 / 1 / c / S]

In this case we consider the problem of :

Locating "n" bins of same item.

Retrieval of bins is one bin at a time.

Only one type of equipment is available (that is, no choice of equipment)

This is the case when all the bins to be stored are of same item, that is, of same type. Retrieval of bins in the warehouse is one bin at a time and equipment is prefixed, that is, there is no need of equipment selection.

Our problem is to locate n bins of same item in the available m locations so that the retrieval cost from these locations is minimized. However as the equipment can store more than one bin at a time (with a limited capacity), the selection of location will also depend on the construction of the tours for the equipment. Thus this is a much more complex problem, where locations and the tours have to be identified simultaneously.

The retrieval cost associated with each location, C_i is given by :

$$C_i = w_2 * f * tr_i \qquad (5.2.1.1)$$

The storage cost will be proportional to the distance traveled by the equipment in all the tours and hence can not be computed for each location separately.

This problem is a generalization of the well known k-Traveling salesman problem, in which a salesman has to select n cities to visit out of possible m cities with a restriction that in each tour he can not visit more than a limited number of cities. The objective is to minimize total cost of such tours which will consist of the cost associated with each city and each link.

A simple heuristic for this problem can be constructed by first identifying the "n" locations optimally with respect to the retrieval cost and then to construct tours using k-TSP heuristics.

5.2.1.1 Heuristic # 1 [HGRD]

In this section we will suggest one heuristic for the problem based on the k-TSP heuristic by Frederickson, Hecht and Kim (1978)[11]

Step I Select "n" locations out of the available "m" locations corresponding to the minimal sum of C_i's.

Step II Apply any TSP heuristic to generate a standard traveling salesman tour. The simplest and appealing method for the TSP is the nearest neighbour algorithm.

(i) Start with a partial tour consisting of a single arbitrarily chosen location(city) a_1.

(ii) If the current partial tour is $a_1, a_2, \ldots, a_k$, $k < n$, Let a_{k+1} be the location, not currently on the tour, which is closest to a_k, and add a_{k+1} to the end of the tour.

(iii) Halt when the current tour contains all the selected locations.

Step III Partition the given tour into subtours. More precisely, suppose that $(i_1, i_2, \ldots, i_n)$ is the order of locations (cities) in the tour T. Include the locations $T_1 = [1, i_2, \ldots, i_{p(1)}]$ in the first subtour depending on the capacity of the equipment. Next subtour $T_2 = [1, i_{p(1)+1}, \ldots i_{p(2)}]$ and $T_k = [1, i_{p(k-1)+1}, \ldots i_n]$.

These is a greedy heuristic which minimizes the retrieval cost in Step I, and the storage cost is minimized in Step II and Step III.

Bellmore and Hong[5] Hong and Padberg[20] and Rao[37] have described other heuristics of k-TSP. These other heuristics can replace the Step II and Step III of the above suggested heuristic.

5.2.2 Case # 2 [n / 1 / k / c / S]

Here we shall consider the problem with the following features:

Locating "n" bins of same item.

Retrieval of bins is one bin at a time.

Choice of equipment is also there.

The problem is same as the problem of case # 1 of 5.2.1, except that best equipment is also to be selected for the selected locations. The problem is to select "n" locations out of "m" locations. The equipment set is composed of several types. Each site can be served by some, but not necessarily all, equipment type. As all the bins are of same item, the equipment set in consideration will be composed of only those equipment's which can serve that item (bin) type.

The three key decisions involved are :

(i) Locations must be selected.

(ii) Equipment's must be assigned to the locations.

(iii) k-TSP routine must be solved for each equipment type.

The retrieval cost computation will be same as that mentioned in case # 1 of 5.2.1 and is given by equation 5.2.1.1.

The simplest heuristic in this case seems to be that first select the optimal locations based on the value of retrieval cost, secondly to optimally assign the selected locations to the available equipment's and in the end solve a k-TSP for each equipment type, so as to optimize the traversed storage distance.

The first and the last steps involved in this heuristic are similar to the heuristic developed in the previous section but to assign the selected locations to the available equipment is a complicated case and our objective in this case may be to reduce the number of equipment's type used as well as optimize the transportation cost.

In the next section we are going to present a greedy heuristic in which a trade off has been made between the number of equipment type assigned and the transportation cost. The k-TSP routine is same as that used for the previous case except that the capacity of each equipment (i.e, size of each tour) may vary.

5.2.2.1 Heuristic # 1 [HGRDEQ]

Step I Select "n" locations out of the available "m" locations corresponding to the minimal sum of C_i's.

Step II For each selected location make a set of type of equipment's which can serve that location. Make a list of locations in the ascending order of the cardinality of this set.

Step III Select the minimum cost equipment for the location on the top of list and delete this location from the list.

Step IV For current location on top of the list see

If

It can be assigned to one of the equipment, which has already been selected for the previous locations.

else

assign minimum cost equipment to the location in consideration.

Step V Delete the assigned location in Step IV from the list and if list is not null then go to Step IV.

Step VI For each equipment type solve the routines of Step VII and Step VIII.

Step VII Apply any TSP heuristic to generate a standard traveling salesman tour. The simplest and appealing method for the TSP is the nearest neighbour algorithm.

(i) Start with a partial tour consisting of a single arbitrarily chosen location(city) a_1.

(ii) If the current partial tour is $a_1, a_2, ..., a_k$, $k < n$, Let a_{k+1} be the location, not currently on the tour, which is closest to a_k, and add a_{k+1} to the end of the tour.

(iii) Halt when the current tour contains all the selected locations.

Step VIII Partition the given tour into subtours More precisely, suppose that $(i_1, i_2, ..., i_n)$ is the order of locations (cities) in the tour T. Include the locations $T_1 = [1, i_2, ..., i_{p(1)}]$ in the first subtour depending on the capacity of the equipment. Next subtour T_2 = $[1, i_{p(1)+1}, ..., i_{p(2)}]$ and $T_k = [1, i_{p(k-1)+1}, ..., i_n]$.

5.2.3 Case # 3 [n / 1 / 1 / c / M]

In this case we consider the problem of :

Locating "n" bins of same item.

Retrieval of bins is more than one bin at a time.

Only one type of equipment is available (that is, no choice of equipment)

This problem is somewhat similar to the case # 3 of section 4.4.3. The retrieval cost computation is same as that given by equation 4.4.3.2. The interaction cost is also same as given by 4.4.3.4. Only the storage cost will .change in this case and will be directly proportional to the distance traveled by the equipment over all the tours.

The retrieval cost is computed as :

$$C_i = f(w_1 tr_i) + w_2 * a_i \qquad (5.2.3.1)$$

So the simplest heuristic in this case seems to be that first select the "n" locations by any of the heuristic in the section 4.4.3.3, 4.4.3.4 and 4.4.3.5 with the modified cost function as given in 5.2.3.1. and then apply the k-TSP heuristic for the selected locations.

In the algorithmic form this heuristic will be given by :

Step I Select "n" locations by any of the heuristic presented in the sections 4.4.3.3-4.4.3.5.

Step II Step II of heuristic 5.2.1.1

Step III Step III of heuristic 5.2.1.1

5.2.4 Case # 4 [n /1 / k / c / M]

Here we shall consider the problem with the following features:

Locating "n" bins of same item.

Retrieval of bins is multi bins at a time.

Choice of equipment is also there.

The problem is same as the problem of case # 3 of 5.2.3, except that best equipment is also to be selected for the selected locations. The retrieval cost associated with each location is computed as in equation 5.2.3.1 .The strategy adopted in this case is exactly similar to that in case # 2 of 5.2.2 except that the initial selection of locations is done by any of the heuristics suggested in the section 4.4.3.3-4.4.3.5 with the modified cost function given by 5.2.3.1. The equipment assignment routine and subrouting routine are same as that of heuristic 5.2.2.1.

5.2.5 Case # 5 [n / b / 1 / c / S]

In this case the problem discussed has the following features :

To locate n bins of b different type of items .

Retrieval of bins is one bin at a time.

Only one type of equipment is available (No choice of equipment is there).

This is the case when we have to locate n bins of b different type of items The number of bins are $n_1, n_2, \ldots\ldots\ldots n_b$ of b items. Here the cost function will comprise of two terms that are a storage cost term which is proportional to the total distance over all the tours and a retrieval cost term which is proportional to the distance of the location from the nearest output dock. The retrieval cost of locating the ith item at jth location is given by :

$$C_{ij} = f_i * [w_1 * tr_j] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.2.5.1)$$

In this problem, as the bins are of different sizes another objective will be to optimize the space utilization of the equipment. This is a separate problem by itself and is referred to as Bin Packing Problem in the literature.

The initial selection of locations can be done on the basis of minimization of the retrieval cost as done in the case # 5 of section 4 4.5 , where the problem is solved as a transportation problem.

After the locations are selected then two strategies can be adopted for subtouring, that is :

(i) Cluster first- Route second.

This strategy is to be adopted when we prioritize the space utilization of the equipment over the tour cost. Here first we have to solve the bin packing problem and then for each equipment solve the TSP. Kraus et al. (1975) [27], introduced an algorithm named first-fit-decrease (FFD) for the bin packing problem. The set of machines is indexed in ascending order. Then, the FFD algorithm orders the set of jobs in descending order of the processing times, and assigns the job on the top of list to the lowest index machine which fit the job. With reference to our problem it means that order the bins in descending order of bin sizes and then go on assigning the bins to an equipment until no other bin can be assigned to that equipment. Delete all the assigned bins from the list and then repeat the whole process for the remaining bins. After that solve TSP for each equipment as in Step II of heuristic 5.2.1.1.

(ii) Route first- Cluster second.

In this case the strategy adopted is same as in the previous cases. The subtours are decided by applying the k-TSP heuristic, as in step II and Step III of heuristic 5.2.1.1.

5.2.6 Case # 6 [n / b / k / c / S]

In this case the problem discussed has following features :

To locate n bins of b different type of items .

Retrieval of bins is one bin at a time.

Choice of equipment is also there.

This problem is same as that of 5 2.5, except that best equipment is also to be selected for the selected locations. Here we shall adopt the cluster first-route second strategy. The locations are selected in the same way as in the previous case of 5.2.5. but the clustering strategy adopted is first-fit-increase(FFD) as suggested by Kraus et al. (1975) [27].The set of machines is indexed in ascending order. Then, the FFD algorithm orders the set of jobs in descending order of the processing times, and assigns the job on the top of list to the lowest index machine which fit the job. With reference to our problem it means that order the bins in descending order of bin sizes and index the equipment's in ascending order of the capacity and then assign the bin on the top of the list to the lowest index machine which may fit that bin. Assign the remaining space to the next bin, if possible and so on. Delete all the assigned bins from the list and then repeat the whole process for the remaining bins. After that solve TSP for each equipment as in Step II of heuristic 5.2.1.1.

5.2.7 Case # 7 [n / b / 1 / c / M]

The problem in this case is :

To locate n bins of b different type of items .

Retrieval of bins is multiple bins at a time.

Only one type of equipment is available (Equipment choice is not there).

This problem is exactly similar to the problem of case # 5 of 5.2.5 except that the retrieval cost in this case will consist of a dissimilarity cost term also. The retrieval cost of locating the ith item at kth location in this case is computed as :

$$C_{ik} = f_i(w_1 * tr_k) + w_2 * a_{ik} \quad \text{...(5.2.7.1)}$$

The initial selection of locations is done by the heuristics 4.4.7.2, 4.4.7.3 and 4.4.7.4 for the Quadratic assignment problem in section 4.4.7.

After the locations are selected the strategy adopted is same as that in case # 5 of 5.2.5 that is either cluster first-route second or route first-cluster second. The heuristics for this are same as in 5.2.5.

5.2.8 Case # 8 [n / b / k / c / M]

In this case the problem discussed has following features :

To locate n bins of b different type of items .

Retrieval of bins is multiple bins at a time.

Choice of equipment is also there.

This is the most general problem of stock assignment in the warehouse. It is same as the problem of case # 7 of 5.2.7, except that best equipment is also to be selected for the selected locations. Here the locations are selected as in the case # 7 of 5.2.7 and after that the strategy adopted is similar to case # 6 of 5.2.6, that is cluster first with the help of FFD algorithm an then solve the Traveling Salesman problem for each equipment.

CHAPTER VI

ORDER PICK UP

6.1 One bin retrieval

The objective while retrieving one bin is simply to minimize the distance traveled for retrieving the bin. So the item to be retrieved should be retrieved from the closest location of that item from the output dock at which the retrieval request is received.

In case equipment selection is also there, we will follow the same strategy as adopted in selecting the equipment while storing the items, that is, equipment's will be associated apriori with the locations and after selecting the nearest location of the item we select the least cost equipment associated with that location. In case the least cost equipment is not available then update the cost of that location with respect to next best available equipment and then again select the nearest location.

6.2 Multiple bin retrieval when equipment can handle one bin at a time

Multiple bin retrieval when equipment can handle only one bin at a time is similar to the case of single bin retrieval. The order of retrieval of the bins does not matters, so any bin can be retrieved any time, keeping in view the due date constraints. So in this case also the objective is to minimize the distance traveled for retrieving the bin. The strategy adopted is to retrieve the item at an output dock from the closest location of that item from the same output dock. For equipment selection also the strategy adopted is same, that is first select the closest item location then for that location select the least cost equipment. In case when least cost equipment is not available for the selected location then choose the next best location.

6.3 Multiple bin retrieval when equipment can handle more than one bin at a time

This is a much more complex problem as compared to the previous cases of 6.1 and 6.2. In this case the objective is to minimize the distance traveled for retrieving the bins over all the tours. So for a retrieval menu the locations from which items are to be retrieved and tours for the equipment to visit these locations are to be identified simultaneously. This is a generalization of the k-TSP and the total retrieval cost will consist of the cost associated with each location and with each link connecting the locations. In the case when choice of equipment is also available the bin packing routine is to be solved in addition to the k-TSP routine. The heuristics for various possible cases are similar to the one which we have discussed in chapter V.

Another complexity can be incorporated in this problem when order clustering is also there, that is, when more then one retrieval requests is received and requests can be clustered together from different orders subject to time restraint, that is, due delivery time of the order. This problem will be equivalent to the generalization of the k-TSP and the combined orders will be treated as a single retrieval menu, with time windows.

CHAPTER VII

CONCLUSIONS AND AVENUES FOR FURTHER WORK

7.1 Conclusions

In this dissertation, the Stock Assignment Problem in an in-process-inventory warehouse has been modeled as a combinatorial optimization problem.

The cases discussed were from the very simple one, that is, to store one bin of an item without any choice of equipment to that of the most general case, that is, to store more than one bin of different items when choice of equipment is present and the equipment's can handle more than one bin at a time.

The resemblance of these problems were found to the standard problems such as Maximum Weighted Clique Problem(MWCP), Time Tabling Problem, Transportation problem, Quadratic Assignment Problem(QAP), Traveling Salesman Problem(TSP) and the k-TSP.

Most of these Problems are NP-Hard. However some efficient heuristics, best among the existing heuristics were suggested for the same. These problems are summarised in Table 7.2 given in the end of this chapter.

7.2 Avenues for further work

Though we suggested some solution methodologies for the problems that we have taken up, it is obvious that further study should be done for these problems to develop better solutions and test them by generating wide variety of test problems.

Logical extension for the current work would be to incorporate Order Picking Problem , which is an integral part of warehouse management. One such problem will be to study the problem of simultaneous selection of locations and tours to optimize the case of retrieval. Preliminary details regarding this problem has been dealt in chapter VI. The variations of this problem is from simple order picking to Batch-order-picking and clustering of orders.

An integrated system comprising of order picking procedures in conjunction with the current work done would address the problem in totality.

S. No.	Problem	Similar Standard problem (if any)	Suggested heuristics
1.	1 / 1 / 1 / 1 / S	------	Select the location with minimum value of Ci.
2.	1 / 1 / k / 1 / S	------	Select the location with minimum value of Ci and the equipment selected apriori / on availability basis
3.	1 / 1 / 1 / 1 / M	------	Same as 1.
4.	1 / 1 / 1 / 1 / M	------	Same as 2.
5.	n / 1 / 1 / 1 / S	------	Select n locations with minimum sum of Ci's.
6.	1 / 1 / k / 1 / S	------	Same as 5. and select the equipments selected apriori / on availability basis
7.	n / 1 / 1 / 1 / M	MWCP & Time Tabling	Heuristic 4.4.3.3, 4.4.3.4, & 4.4.3.5.
8.	n / 1 / k / 1 /M	MWCP & Time Tabling	Same as 7. and select the equipments selected apriori / on availability basis.
9.	n / b / 1 / 1 / S	Transportation Problem	Heuristic 4.4.5.2(VAM) & 4.4.5.3.(RAM)
10.	n / b / k / 1 / S	Transportation Problem	Heuristic 4.4.6.1 (VAM)

Table 7.2 continued...

11.	n / b / 1 / 1 / M	Quadratic Assignment Problem	Heuristic 4.4.7.2(VAM) & 4.4.7.3 (CRAFT)
12.	n / b / k / 1 / M	Quadratic Assignment Problem	Heuristic 4.4.8.1. (VAM)
13.	n / 1 / 1 / c / S	k-TSP	Heuristic 5.2.1.1 (nearest neighbour + tour partitioning)
14.	n / 1 / k / c / S	k-TSP	Heuristic 5.2.2.1. (nearest neighbour + tour partitioning)
15.	n / 1 / 1 / c / M	k-TSP + MWCP	Same as 13.
16.	n / 1 / k / c / M	k-TSP + MWCP	Same as 14.
17.	n / b / 1 / c / S	Transportation + Bin packing + k-TSP	VAM + FFD + nearest neighbour + tour partitioning.
18.	n / b / k / c / S	Transportation + Bin packing + k-TSP	VAM + FFD + nearest neighbour + tour partitioning.
19.	n / b / 1 / c / M	QAP + Bin packing + k-TSP	Heuristic 4.4.7.2 + FFD + 5.2.1.1
20.	n / b / k / c / M	QAP + Bin packing + k-TSP	Heuristic 4.4.7.2 + FFD + 5.2.2.1.

Table 7.2 : Summary of the problems dealt in the dissertation.

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